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2002 J. Phys. A: Math. Gen. 35 L753

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INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) L753–L758 PII: S0305-4470(02)54715-6

LETTER TO THE EDITOR

Large distance asymptotic behaviour of the emptiness formation probability of the XXZ spin-$\frac{1}{2}$ Heisenberg chain

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Received 11 August 2002
Published 28 November 2002
Online at stacks.iop.org/JPhysA/35/L753

Abstract
Using its multiple integral representation, we compute the large distance asymptotic behaviour of the emptiness formation probability of the XXZ spin-$\frac{1}{2}$ Heisenberg chain in the massless regime.

PACS numbers: 71.45.G, 75.10.Jm, 11.30.Na, 03.65.Fd

1. Emptiness formation probability at large distance

The Hamiltonian of the XXZ spin-$\frac{1}{2}$ Heisenberg chain is given by

$$H = \sum_{m=1}^{M} \left(\sigma_m^x \sigma_{m+1}^x + \sigma_m^y \sigma_{m+1}^y + \Delta\left(\sigma_m^z \sigma_{m+1}^z - 1\right)\right). \tag{1.1}$$

Here Δ is the anisotropy parameter, and $\sigma_m^{x,y,z}$ denote the usual Pauli matrices acting on the quantum space at site m of the chain. The emptiness formation probability $\tau(m)$ (the probability of finding in the ground state a ferromagnetic string of length m) is defined as the following expectation value

$$\tau(m) = \langle \psi_g | \prod_{k=1}^{m} \frac{1 - \sigma_k^z}{2} | \psi_g \rangle \tag{1.2}$$

where $|\psi_g\rangle$ denotes the normalized ground state.

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The methods based on the q-vertex operator approach [1–3] and the algebraic Bethe ansatz [4, 5] allow us to express the general correlation function of this model, in the thermodynamic limit ($M \to \infty$), as multiple integrals. In this framework, the emptiness formation probability $\tau(m)$ of the XXZ chain is given as an integral with m integrations. The limit $\Delta \to 1$ (XXX case) of the representation for $\tau(m)$ following from [1] was given in [6]. The first evaluation of the multiple integral for the emptiness formation probability with arbitrary m has been performed in [7] for the special case $\Delta = 0$ (free fermions).

Recently, in [8], a new multiple integral representation for $\tau(m)$ was obtained. It leads in a direct way to the above-mentioned answer at $\Delta = 0$ [9], in particular using a saddle-point method. This new representation also allows us to obtain the first exact result for the emptiness formation probability (for arbitrary m) outside the free fermion point (namely at $\Delta = \frac{1}{2}$) [10], a formula first conjectured in [11].

The purpose of this letter is to present an analytical evaluation of the asymptotic behaviour of $\tau(m)$ at large distance m, in the massless regime $-1 < \Delta < 1$, via the saddle-point method applied to the new multiple integral representation [8]. We find

$$\lim_{m\to\infty} \frac{\log \tau(m)}{m^2} = \log \frac{\pi}{\zeta} + \frac{1}{2} \int_{\mathbb{R}-\mathrm{i}0} \frac{\mathrm{d}\omega}{\omega} \frac{\sinh \frac{\omega}{2}(\pi - \zeta) \cosh^2 \frac{\omega\zeta}{2}}{\sinh \frac{\pi\omega}{2} \sinh \frac{\omega\zeta}{2} \cosh \omega\zeta} \tag{1.3}$$

where $\cos\zeta = \Delta, 0 < \zeta < \pi$. If ζ is commensurate with π (in other words if $\mathrm{e}^{\mathrm{i}\zeta}$ is a root of unity), then the integral in equation (1.3) can be taken explicitly in terms of the ψ-function (a logarithmic derivative of the Γ-function). In particular, for $\zeta = \frac{\pi}{2}$ and $\zeta = \frac{\pi}{3}$ (respectively $\Delta = 0$ and $\Delta = 1/2$) we obtain from equation (1.3)

$$\begin{aligned} \lim_{m\to\infty} \frac{\log \tau(m)}{m^2} &= -\frac{1}{2} \log 2 \qquad && \Delta = 0 \\ \lim_{m\to\infty} \frac{\log \tau(m)}{m^2} &= \frac{3}{2} \log 3 - 3 \log 2 \qquad && \Delta = \frac{1}{2} \end{aligned} \tag{1.4}$$

which coincides with the exact known results obtained respectively in [7, 9, 12] and in [10, 11]. For the particular case of the XXX chain ($\Delta = 1, \zeta = 0$) the asymptotic behaviour can also be evaluated by the saddle-point method and it is given by

$$\lim_{m\to\infty} \frac{\log \tau(m)}{m^2} = \log \left(\frac{\Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}\right)} \right) \approx \log(0.5991) \tag{1.5}$$

which is in good agreement with the numerical result log(0.598), obtained in [13].

In the following, we explain the main features of our method. A more detailed account of the proofs and techniques involved will be published later.

2. The saddle-point method

The multiple integral representation for $\tau(m)$ obtained in [8] can be written in the form

$$\begin{aligned} \tau(m) = {} & \left(\frac{\mathrm{i}}{2\zeta \sin\zeta}\right)^m \left(\frac{\pi}{\zeta}\right)^{\frac{m^2-m}{2}} \int_{\mathcal{D}} \mathrm{d}^m\lambda \, F(\{\lambda\}, m) \\ & \times \prod_{a>b}^{m} \frac{\sinh \frac{\pi}{\zeta}(\lambda_a - \lambda_b)}{\sinh(\lambda_a - \lambda_b - \mathrm{i}\zeta)\sinh(\lambda_a - \lambda_b + \mathrm{i}\zeta)} \\ & \times \prod_{a=1}^{m} \left(\frac{\sinh\left(\lambda_a - \frac{\mathrm{i}\zeta}{2}\right)\sinh\left(\lambda_a + \frac{\mathrm{i}\zeta}{2}\right)}{\cosh \frac{\pi}{\zeta}\lambda_a} \right)^m \end{aligned} \tag{2.1}$$

with

$$F(\{\lambda\},m)=\lim_{\xi_1,\ldots,\xi_m\to-\frac{i\zeta}{2}}\frac{1}{\prod_{a>b}^{m}\sinh(\xi_a-\xi_b)}\det_m\left(\frac{-\mathrm{i}\sin\zeta}{\sinh(\lambda_j-\xi_k)\sinh(\lambda_j-\xi_k-\mathrm{i}\zeta)}\right). \tag{2.2}$$

Here the integration domain $\mathcal{D}$ is $-\infty<\lambda_1<\lambda_2<\cdots<\lambda_m<\infty$.

To estimate the integral equation (2.1) we use the saddle-point method. Although this method is not completely rigorous from the mathematical point of view, it is widely used and is known to give sensible answers. Moreover, as already discussed in section 1, it will give in our case an explicit formula in complete agreement with the exact known answers for $\zeta=\frac{\pi}{2}$ and $\zeta=\frac{\pi}{3}$ (respectively $\Delta=0$ and $\Delta=1/2$).

Following the standard arguments of the saddle-point approach we estimate the integral in equation (2.1) by the maximal value of the integrand. Let $\{\lambda'\}$ be the set of parameters corresponding to this maximum. They satisfy the saddle-point equations and for large m we assume that their distribution can be described by a density function $\rho(\lambda')$:

$$\rho(\lambda'_j)=\lim_{m\to\infty}\frac{1}{m(\lambda'_{j+1}-\lambda'_j)}. \tag{2.3}$$

Thus for large m, we can replace sums over the set $\{\lambda'\}$ by integrals. Namely, if $f(\lambda)$ is integrable on the real axis, then

$$\begin{aligned}&\frac{1}{m}\sum_{j=1}^{m}f(\lambda'_j)\to\int_{-\infty}^{\infty}f(\lambda)\rho(\lambda)\,\mathrm{d}\lambda\\&\frac{1}{m}\sum_{\substack{j=1\\j\neq k}}^{m}\frac{f(\lambda'_j)}{\lambda'_j-\lambda'_k}\to\mathrm{V.P.}\int_{-\infty}^{\infty}\frac{f(\lambda)}{\lambda-\lambda'_k}\rho(\lambda)\,\mathrm{d}\lambda\end{aligned}\qquad m\to\infty. \tag{2.4}$$

Due to equation (2.4) it is easy to see that in the point $\lambda'_1,\ldots,\lambda'_m$ the products in the second line of equation (2.1) behave as $\exp(cm^2)$.

Our goal is now to estimate the behaviour of the term $F(\{\lambda'\},m)$. To do this we factorize the determinant in equation (2.2) as follows for large m

$$\det_m\left(\frac{-\mathrm{i}\sin\zeta}{\sinh(\lambda'_j-\xi_k)\sinh(\lambda'_j-\xi_k-\mathrm{i}\zeta)}\right)\longrightarrow(-2\pi\,\mathrm{i})^m\det_m\left(\delta_{jk}-\frac{K(\lambda'_j-\lambda'_k)}{2\pi\,\mathrm{i}m\rho(\lambda'_k)}\right)\det_m\left(\frac{\mathrm{i}}{2\zeta\sinh\frac{\pi}{\zeta}(\lambda'_j-\xi_k)}\right) \tag{2.5}$$

with

$$K(\lambda)=\frac{-\mathrm{i}\sin 2\zeta}{\sinh(\lambda-\mathrm{i}\zeta)\sinh(\lambda+\mathrm{i}\zeta)}. \tag{2.6}$$

Indeed, for $m\to\infty$ and $-\zeta<\mathrm{Im}\,\xi_k<0$ we have

$$\begin{aligned}&\det_m\left(\delta_{jk}-\frac{K(\lambda'_j-\lambda'_k)}{2\pi\,\mathrm{i}m\rho(\lambda'_k)}\right)\det_m\left(\frac{\mathrm{i}}{2\zeta\sinh\frac{\pi}{\zeta}(\lambda'_j-\xi_k)}\right)\\&\quad=\det_m\left(\frac{\mathrm{i}}{2\zeta\sinh\frac{\pi}{\zeta}(\lambda'_j-\xi_k)}-\sum_{l=1}^{m}\frac{K(\lambda'_j-\lambda'_l)}{2\pi\,\mathrm{i}m\rho(\lambda'_l)}\frac{\mathrm{i}}{2\zeta\sinh\frac{\pi}{\zeta}(\lambda'_l-\xi_k)}\right)\\&\quad\longrightarrow\det_m\left(\frac{\mathrm{i}}{2\zeta\sinh\frac{\pi}{\zeta}(\lambda'_j-\xi_k)}-\int_{-\infty}^{\infty}\frac{K(\lambda'_j-\mu)}{2\pi\,\mathrm{i}}\frac{\mathrm{i}\,\mathrm{d}\mu}{2\zeta\sinh\frac{\pi}{\zeta}(\mu-\xi_k)}\right)\\&\quad=\left(\frac{1}{2\pi}\right)^m\det_m\left(\frac{\sin\zeta}{\sinh(\lambda'_j-\xi_k)\sinh(\lambda'_j-\xi_k-\mathrm{i}\zeta)}\right).\end{aligned} \tag{2.7}$$

Here we have used the fact that the function $\mathrm{i}/2\zeta \sinh \frac{\pi}{\zeta}(\lambda_j - \xi)$ solves the Lieb integral equation for the density of the ground state of the XXZ magnet [14] (and we have used the notations of [8]). The second determinant on the right-hand side of equation (2.5) is a Cauchy determinant, hence

$$F(\{\lambda'\}, m) = (-\mathrm{i})^m \left(\frac{\pi}{\zeta}\right)^{\frac{m^2+m}{2}} \frac{\prod_{a>b}^m \sinh \frac{\pi}{\zeta}(\lambda'_a - \lambda'_b)}{\prod_{a=1}^m \cosh^m \frac{\pi}{\zeta}\lambda'_a} \det_m \left(\delta_{jk} - \frac{K(\lambda'_j - \lambda'_k)}{2\pi \mathrm{i} m \rho(\lambda'_k)}\right). \tag{2.8}$$

The behaviour of the determinant in equation (2.8) can be estimated via the Hadamard inequality

$$|\det_m(a_{jk})| \leqslant (\max |a_{jk}|)^m m^{\frac{m}{2}} \tag{2.9}$$

applied to the above determinant and to the determinant of the inverse matrix, which shows that

$$\lim_{m\to\infty} \frac{1}{m^2} \log \det_m \left(\delta_{jk} - \frac{K(\lambda'_j - \lambda'_k)}{2\pi \mathrm{i} m \rho(\lambda'_k)}\right) = 0. \tag{2.10}$$

The previous equation means that $\det_m(\delta_{jk} - K(\lambda'_j - \lambda'_k)/2\pi \mathrm{i} m\rho(\lambda'_k))$ does not contribute to the leading term of the asymptotic behaviour. Hence, it can be excluded from our present considerations. Note however that this determinant will contribute to the sub-leading corrections.

Thus, the emptiness formation probability behaves as

$$\tau(m) \longrightarrow \left(\frac{\pi}{\zeta}\right)^{m^2} \mathrm{e}^{m^2 S_0 + o(m^2)} \qquad m \to \infty \tag{2.11}$$

with

$$\begin{aligned} S_0 \equiv S(\{\lambda'\}) = {} & \frac{1}{m^2} \sum_{a>b}^m \log \left(\frac{\sinh^2 \frac{\pi}{\zeta}(\lambda'_a - \lambda'_b)}{\sinh(\lambda'_a - \lambda'_b - \mathrm{i}\zeta) \sinh(\lambda'_a - \lambda'_b + \mathrm{i}\zeta)} \right) \\ & + \frac{1}{m} \sum_{a=1}^m \log \left(\frac{\sinh(\lambda'_a - \mathrm{i}\zeta/2) \sinh(\lambda'_a + \mathrm{i}\zeta/2)}{\cosh^2 \frac{\pi}{\zeta}\lambda'_a} \right). \end{aligned} \tag{2.12}$$

Here the parameters $\{\lambda'\}$ are the solutions of the saddle-point equations

$$\frac{\partial S_0}{\partial \lambda'_j} = 0. \tag{2.13}$$

In our case the system equation (2.13) has the form

$$\begin{aligned} & \frac{2\pi}{\zeta} \tanh \frac{\pi \lambda'_j}{\zeta} - \coth(\lambda'_j - \mathrm{i}\zeta/2) - \coth(\lambda'_j + \mathrm{i}\zeta/2) \\ & \quad = \frac{1}{m} \sum_{\substack{k=1 \\ k\neq j}}^m \left(\frac{2\pi}{\zeta} \coth \frac{\pi}{\zeta}(\lambda'_j - \lambda'_k) - \coth(\lambda'_j - \lambda'_k - \mathrm{i}\zeta) - \coth(\lambda'_j - \lambda'_k + \mathrm{i}\zeta) \right). \end{aligned} \tag{2.14}$$

Using equation (2.4) we transform equation (2.14) into the integral equation for the density $\rho(\lambda)$

$$\begin{aligned} & \frac{2\pi}{\zeta} \tanh \frac{\pi \lambda}{\zeta} - \coth(\lambda - \mathrm{i}\zeta/2) - \coth(\lambda + \mathrm{i}\zeta/2) \\ & \quad = \mathrm{V.P.} \int_{-\infty}^{\infty} \left(\frac{2\pi}{\zeta} \coth \frac{\pi}{\zeta}(\lambda - \mu) - \coth(\lambda - \mu - \mathrm{i}\zeta) \right. \\ & \qquad \left. - \coth(\lambda - \mu + \mathrm{i}\zeta) \right) \rho(\mu)\, \mathrm{d}\mu. \end{aligned} \tag{2.15}$$

Respectively, the action S_0 takes the form

$$S_0 = \int_{-\infty}^{\infty} \mathrm{d}\lambda \rho(\lambda) \log\left(\frac{\sinh(\lambda - \mathrm{i}\zeta/2)\sinh(\lambda + \mathrm{i}\zeta/2)}{\cosh^2 \frac{\pi}{\zeta}\lambda}\right)$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}\mu\, \mathrm{d}\lambda \rho(\lambda)\rho(\mu) \log\left(\frac{\sinh^2 \frac{\pi}{\zeta}(\lambda - \mu)}{\sinh(\lambda - \mu - \mathrm{i}\zeta)\sinh(\lambda - \mu + \mathrm{i}\zeta)}\right). \tag{2.16}$$

Since the kernel of the integral operator in equation (2.15) depends on the difference of the arguments, this equation can be solved via Fourier transform. Then

$$\hat{\rho}(\omega) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega\lambda}\rho(\lambda)\,\mathrm{d}\lambda = \frac{\cosh\frac{\omega\zeta}{2}}{\cosh\omega\zeta}. \tag{2.17}$$

Performing the inverse Fourier transform we find

$$\rho(\lambda) = \frac{\cosh\frac{\pi\lambda}{2\zeta}}{\zeta\sqrt{2}\cosh\frac{\pi\lambda}{\zeta}} \tag{2.18}$$

which obviously satisfies the necessary normalization condition for density (the integral on the real axis equals one). It remains to substitute equations (2.17) and (2.18) into equation (2.16), and after straightforward calculations we arrive at

$$S_0 = \frac{1}{2}\int_{\mathbb{R}-\mathrm{i}0} \frac{\mathrm{d}\omega}{\omega}\frac{\sinh\frac{\omega}{2}(\pi - \zeta)\cosh^2\frac{\omega\zeta}{2}}{\sinh\frac{\pi\omega}{2}\sinh\frac{\omega\zeta}{2}\cosh\omega\zeta}. \tag{2.19}$$

Thus, we have obtained equation (1.3).

In the case of the XXX chain ($\Delta = 1$) we should rescale $\lambda_j \to \zeta\lambda_j$, $\xi_j \to \zeta\xi_j$ in the original multiple integral representation equation (2.1) for $\tau(m)$ and then proceed to the limit $\zeta \to 0$. The remaining computations are then very similar to those described above, therefore we present here only the main results. The behaviour of $\tau(m)$ is now given by

$$\tau(m) \longrightarrow \pi^{m^2}\mathrm{e}^{m^2 S_0 + o(m^2)} \qquad m \to \infty. \tag{2.20}$$

The action S_0 in the saddle point has the form

$$S_0 = \int_{-\infty}^{\infty} \log\left(\frac{(\lambda - \mathrm{i}/2)(\lambda + \mathrm{i}/2)}{\cosh^2\pi\lambda}\right)\rho(\lambda)\,\mathrm{d}\lambda$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}\mu\,\mathrm{d}\lambda\rho(\lambda)\rho(\mu)\log\left(\frac{\sinh^2\pi(\lambda - \mu)}{(\lambda - \mu - \mathrm{i})(\lambda - \mu + \mathrm{i})}\right). \tag{2.21}$$

The analogue of the integral equation (2.15) in the XXX case is

$$2\pi\tanh\pi\lambda - \frac{2\lambda}{\lambda^2 + \frac{1}{4}} = \mathrm{V.P.}\int_{-\infty}^{\infty}\left(2\pi\coth\pi(\lambda - \mu) - \frac{2(\lambda - \mu)}{(\lambda - \mu)^2 + 1}\right)\rho(\mu)\,\mathrm{d}\mu. \tag{2.22}$$

The solution of this equation is

$$\rho(\lambda) = \frac{\cosh\frac{\pi\lambda}{2}}{\sqrt{2}\cosh\pi\lambda}. \tag{2.23}$$

Substituting equation (2.23) into equation (2.21) we finally arrive at equation (1.5).

In conclusion we would like to mention that in this letter we have focused our attention only on the rate of the Gaussian decay of the emptiness formation probability and we have not discussed the corrections to this driving term of the asymptotic behaviour. Nevertheless, according to our preliminary analysis, the saddle-point method seems also to be fruitful for the evaluation of such sub-leading terms. Such corrections come from various sources including the determinant in equation (2.8), the usual saddle-point corrections due to the determinant of the matrix of second derivatives of S_0, and also from the replacement of sums with integrals. This question is now being studied.

After we submitted this letter to the hep-th preprint archive [15], another paper appeared [16] also dealing with the asymptotic behaviour of the emptiness formation probability $\tau(m)$. There the authors conjectured, in addition to the same Gaussian decay described in our work, a formula for the exponent giving a power-law correction to the above asymptotic behaviour of $\tau(m)$, these predictions being supported by numerical simulations (the Density Matrix Renormalization Group (DMRG) and Monte Carlo methods).

Acknowledgments

NK is supported by JSPS grant P01177. NK would like to thank M Jimbo for help. NS is supported by the grants RFBR-02-01-00484, Foundation of the Support of Russian Science, Leading Scientific Schools 00-15-96046, the Programme of Nonlinear Dynamics and Solitons and by CNRS. VT is supported by DOE grant DE-FG02-96ER40959. JMM and VT are supported by CNRS and by the European Community contract HPRN-CT-2002-00325. NK, NS and VT would like to thank the Theoretical Physics group of the Laboratory of Physics at ENS Lyon for hospitality, which makes this collaboration possible. We also would like to thank the organizers of the 6th International Workshop Conformal Field Theory and Integrable Models held in Chernogolovka, 15–21 September 2002, for the stimulating scientific (and extra-scientific) atmosphere they succeeded in generating.

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